# MATH 54-MOCK MIDTERM 1 

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Name:
Instructions: This is a mock midterm, designed to give you extra practice for the actual midterm. Good luck!!!

| 1 |  | 25 |
| :--- | :--- | ---: |
| 2 |  | 15 |
| 3 |  | 20 |
| 4 |  | 10 |
| 5 |  | 10 |
| 6 |  | 10 |
| 7 |  | 10 |
| Total |  | 100 |

1. (25 points, 5 pts each)

Label the following statements as $\mathbf{T}$ or $\mathbf{F}$.
Make sure to JUSTIFY YOUR ANSWERS!!! You may use any facts from the book or from lecture.
(a) If $A$ and $B$ are square matrices, then $(A+B)^{-1}=A^{-1}+B^{-1}$.
(b) If $T: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$ is a one-to-one linear transformation, then $T$ is also onto.
(c) If $\left\{\mathbf{v}_{\mathbf{1}}, \mathbf{v}_{\mathbf{2}}, \mathbf{v}_{\mathbf{3}}\right\}$ are linearly independent vectors in $\mathbb{R}^{n}$, then $\left\{\mathbf{v}_{\mathbf{1}}, \mathbf{v}_{\mathbf{2}}\right\}$ is linearly independent as well!
(d) If $A$ is an invertible square matrix, then $\left(A^{T}\right)^{-1}=\left(A^{-1}\right)^{T}$
(e) If $A$ is a $3 \times 3$ matrix with two pivot positions, then the equation $A \mathbf{x}=\mathbf{0}$ has a nontrivial solution.
2. (15 points) Solve the following system (or say it has no solutions):

$$
\left\{\begin{array}{c}
x+y+z=0 \\
2 x+2 z=0 \\
3 x+y+3 z=0
\end{array}\right.
$$

3. (20 points) Find the inverse of the following matrix:

$$
A=\left[\begin{array}{ccc}
1 & -1 & 1 \\
1 & 1 & 2 \\
1 & 0 & 1
\end{array}\right]
$$

4. (10 points) What's the next elementary row operation you would use to transform the following matrix in row-echelon form? What is the corresponding elementary matrix?

$$
A=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & -1 \\
0 & 2 & 1
\end{array}\right]
$$

5. (10 points, 5 points each) Evaluate the following products if they are defined, or say 'undefined'
(a) $A B$, where:

$$
A=\left[\begin{array}{cc}
2 & 5 \\
0 & 7 \\
-1 & 3
\end{array}\right], B=\left[\begin{array}{l}
1 \\
0
\end{array}\right]
$$

(b) $A B$, where:

$$
A=\left[\begin{array}{ccc}
1 & 0 & 1 \\
0 & 1 & 0 \\
2 & -1 & 0
\end{array}\right], B=\left[\begin{array}{lll}
0 & 1 & 0 \\
2 & 1 & 3 \\
0 & 0 & 1
\end{array}\right]
$$

6. (10 points) Suppose $T: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ is a linear transformation which reflects points in the plane about the origin.
(a) (5 points) Find the matrix $A$ of $T$.
(b) (5 points) Use $A$ to find $T(1,1)$.
7. (10 points) Find a basis for $\operatorname{Nul}(A)$ and $\operatorname{Col}(A)$, where $A$ is the following matrix:

$$
A=\left[\begin{array}{ccc}
1 & 1 & 3 \\
0 & -1 & 1 \\
0 & 1 & 2
\end{array}\right]
$$

